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THE SENSITIVITY OF THIN-WALLED COMPRESSION MEMBERS TO COLUMN AXIS IMPERFECTION

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Abstract—Imperfection sensitivity of thin-walled compression members to initial waviness of the composing walls and to imperfection of the column axis was demonstrated in Ref. [1]. The effect of the latter imperfection however was not adequately analyzed. The present report gives the correction and evaluates in the case of an idealized column the reduction of the failure load due to this kind of imperfection, which appears to be significant. Elastic behaviour has been assumed.

NOTATION

a	wave amplitude of flange imperfection
b	flange width (Fig. 1)
c	half of section depth, radius of gyration (Fig. 1)
e_0	wave amplitude of column axis imperfection
h	flange thickness (Fig. 1)
x	longitudinal coordinate
I	inertia moment of column section
K	compressive load of column, failure load
L	half wavelength of overall buckling
M	bending moment
P	flange load
R	$K_E/K_l = 3(1-\nu^2)(b/h \,.\, c/L)^2$, interaction parameter
W	deflexion of column
α	a/h, flange imperfection parameter
β	e_0/c, column axis imperfection parameter
ε	edge strain of flange
η	$d(P/P_l)/d(\varepsilon/\varepsilon_l)$
λ	K/K_l, load parameter
μ	$\frac{1}{6}(2\eta'^2 - \eta\eta'')$
ω	W/c
ξ	$\pi x/L$
$(\ )'$	$d(\ )/d(P/P_l)$
$(\ )^{\cdot}$	$d(\ )/d\xi$

suffix b refers to buckling load for $\alpha \neq 0$, $\beta = 0$
suffix l refers to local buckling load
suffix E refers to Euler buckling load

1. INTRODUCTION

THE author's first paper [1] on the interaction of local buckling and overall buckling of thin-walled compression members showed in the case of an idealized column severe imperfection-sensitivity when the ratio R of Euler load K_E to local buckling load K_l is about one. Two kinds of imperfection were considered; 1. imperfections similar to the local buckling mode, the amplitude of which is α times the wall thickness; 2. imperfections similar to the overall buckling mode.

It was shown that the imperfection α, irrespective of its magnitude, keeps the buckling load K_b below the local buckling load K_l, for the considered model up to $R = 2$. Subsequently Koiter and van Kuiken [2] gave asymptotic formulae for the strength reduction due to α and Thompson [3] investigated on the basis of the data of [1] the optimum design, showing that the commonly accepted criterion that the optimum is at $R = 1$ is wrong and that the optimum shifts to some value of $R < 1$ depending on α and erodes more and more with α increasing.

In order to investigate the sensitivity to column axis imperfection Ref. [1] established the slope of the load–deflexion curve at the bifurcation K_b. It appeared that the equilibrium at K_b is instable with respect to small finite deflexions up to $R = 1{\cdot}4$–$1{\cdot}6$, depending on α. Therefore column imperfection-sensitivity could be expected. Reference [1] also deals with the further strength reduction due to this effect. However the author upon further examination of this latter problem has come to the conclusion that this part of Ref. [1] needs correction. The object of the present paper is to bring the correction and to discuss in more detail the interesting and by no means negligible effect of column axis imperfection. Though the nonlinear relation between the bending moment M and small finite deflexion W of the column as given in Ref. [1] is correct, its derivation is not quite straightforward. The Appendix of this paper gives a more elegant derivation. This derivation has been given in an earlier paper by Meyer and van der Neut [4], which got only limited distribution.

2. THE DIFFERENTIAL EQUATION AND ITS SOLUTION

The model considered in Ref. [1] has the section shown in Fig. 1. The axial stress carrying parts are two equal flanges (width b, thickness h) with equal imperfection α, simply supported at their edges by webs (width $2c$) which serve to maintain the structural integrity of the strut without contributing to the transmission of axial stresses.

Due to eccentricity $e(x)$ of the column axis any axial load K causes bending moments and consequently finite deflexions $W(x)$.

The equilibrium of the element $\mathrm{d}x$ requires

$$\frac{\mathrm{d}^2 M}{\mathrm{d}x^2} + K\frac{\mathrm{d}^2(W+e)}{\mathrm{d}x^2} = 0. \tag{1}$$

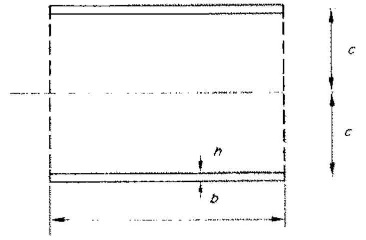

FIG. 1.

M can be expressed in W by means of the equation (6) of the Appendix with these two alterations:

1. η and its derivatives pertaining to K_b have to be replaced by these quantities pertaining to K;
2. $k = 0$.

Then

$$M = \eta EI\left[\frac{d^2W}{dx^2} - \mu\left(\frac{c}{\varepsilon_l}\right)^2\left(\frac{d^2W}{dx^2}\right)^3\right], \tag{2}$$

where

$$\mu = \tfrac{1}{6}(2\eta'^2 - \eta\eta''). \tag{3}$$

One might have some doubt about the validity of Appendix, equation (6) for this case where the column axis is imperfect in the unloaded condition. The proof of its validity has been given in the following way, where the final deflected configuration W is being reached through a number of stages. The complete derivation will not be reproduced because it is rather lengthy.

Stage 0. The column is unloaded in its initially curved position, $\varepsilon_{10} = \varepsilon_{20} = 0$ (upper and lower flange will be indicated by the suffices 1 and 2 respectively; the stage number appears as second suffix).

Stage 1. The load K is applied and deflexion is being prevented ($W_1 = 0$); $\varepsilon_{11} = \varepsilon_{21} = \varepsilon_K$ and $P_{11} = P_{21} = P_K = \frac{1}{2}K$.

Stage 2. While the load K is still present the deflexion $W_2 = -e$ is being forced upon the column, straightening it. The edge strains ε_{12}, $\varepsilon_{22} = \varepsilon_{12} + 2c(d^2e/dx^2)$ and the flange loads P_{12}, $P_{22} = K - P_{12}$ are unknown.

Stage 3. The column is brought back to stage 1 by the deflexion $W_3 = e$. Hence $\varepsilon_{i3} = \varepsilon_K$ and $P_{i3} = \frac{1}{2}K$ ($i = 1, 2$). Using the Taylor series of P in the vicinity of P_{i2} (up to the third order term), P_K ($=\frac{1}{2}K$) and the first, second and third derivatives of P to ε at P_K can be expressed into $\varepsilon_K - \varepsilon_{i2}$, P_{i2} and the derivatives at P_{i2}. Inversely these equations can be used to solve for P_{i2} and the derivatives at P_{i2}, expressing them into $\varepsilon_K - \varepsilon_{i2}$, P_K and the derivatives at P_K.

The condition $P_{12} + P_{22} = K$ yields $\varepsilon_{12} - \varepsilon_K$ (and $\varepsilon_{22} - \varepsilon_K$).

Stage 4. The column is brought back to Stage 2 by the deflexion $W_4 = -e$, straightening it again.

Stage 5. Starting from Stage 4 with straight column axis the column is brought in its final position by the deflexion $W_5 = W + e$. Applying again the Taylor series of P, which has been used in Stage 3, the final flange loads $P_{i5} = P_i$ are being expressed into P_{i2} and the derivatives at P_{i2}, which have been established at Stage 3, and into $\varepsilon_{i5} - \varepsilon_{i2} = \varepsilon_i - \varepsilon_{i2}$. Using $\varepsilon_2 = \varepsilon_1 - 2c[d^2(W+e)/dx^2]$ the condition $P_1 + P_2 = K$ yields $\varepsilon_1 - \varepsilon_{12}$. Next P_1 and P_2 can be established, which yield equation (2) for the bending moment $M = (P_1 - P_2)c$.

The nonlinear differential equation of W appears to be

$$\frac{d^2}{dx^2}\left\{\eta EI\left[\frac{d^2W}{dx^2} - \mu\left(\frac{c}{\varepsilon_l}\right)^2\left(\frac{d^2W}{dx^2}\right)^3\right] + K(W+e)\right\} = 0. \tag{4}$$

This equation will be solved for the column of length $2L$ clamped at its ends, then the boundary conditions are

$$x = \pm L : W = 0, \qquad \frac{dW}{dx} = 0, \tag{5}$$

and with the imperfection corresponding to the Euler mode

$$e = e_0(\cos \pi x/L + 1). \tag{6}$$

With the dimensionless coordinate $\xi = \pi x/L$, the load ratios $R = K_E/L_l$ and $\lambda = K/K_l$, the dimensionless deflexion $\omega = W/c$, the imperfection parameter $\beta = e_0/c$ and taking into account that $\pi^2 c^2/L^2 = \varepsilon_E$ equations (4–6) become

$$\left[\eta R(\ddot{\omega} - \mu R^2 \ddot{\omega}^3) + \lambda\left(\omega + \beta \frac{e}{e_0}\right)\right]^{\cdot\cdot} = 0, \tag{7}$$

$$\xi = \pm\pi : \omega = 0, \qquad \dot{\omega} = 0, \tag{8}$$

$$\frac{e}{c} = \beta(\cos \xi + 1), \tag{9}$$

where $(\ )^{\cdot} = [d(\ )/d\xi]$.

The boundary conditions are satisfied by a solution

$$\omega = \sum_{i=1}^{\infty} \omega_{2i-1}[\cos(2i-1)\xi + 1]. \tag{10}$$

Integrating equation (7) twice:

$$\ddot{\omega} - \mu R^2 \ddot{\omega}^3 + \frac{\lambda}{\eta R}\omega = -\frac{\lambda}{\eta R}\beta(\cos \xi + 1) + \text{constant}. \tag{11}$$

Restricting equation (10) to

$$\omega = \omega_1 \cos \xi + \omega_3 \cos 3\xi + \omega_5 \cos 5\xi + (\omega_1 + \omega_3 + \omega_5) \tag{12}$$

substitution of ω into equation (11), thereby neglecting the cos 7ξ-terms, etc. yields

$$\frac{\lambda}{\eta R}(\omega_1 + \omega_2 + \omega_3) = \frac{\lambda}{\eta R}\beta + \text{constant},$$

$$\left[-\left(1 - \frac{\lambda}{\eta R}\right)\omega_1 + \tfrac{3}{4}\mu R^2 \omega_1^3 \cdot \phi_1\right] \cos \xi = -\frac{\lambda}{\eta R}\beta \cos \xi, \tag{13}$$

$$\left[-\left(9 - \frac{\lambda}{\eta R}\right)\omega_3 + \tfrac{1}{4}\mu R^2 \omega_1^3 \phi_3\right] \cos 3\xi = 0, \tag{14}$$

$$\left[-\left(25 - \frac{\lambda}{\eta R}\right)\omega_5 + \tfrac{27}{4}\mu R^2 \omega_1^2 \omega_3 \phi_5\right] \cos 5\xi = 0, \tag{15}$$

where

$$\phi_1 = 1+9\frac{\omega_3}{\omega_1}+2.9^2\left(\frac{\omega_3}{\omega_1}\right)^2+2.9.25\frac{\omega_3\omega_5}{\omega_1^2}+2.25^2\left(\frac{\omega_5}{\omega_1}\right)^2+9^2.25\frac{\omega_3^2\omega_5}{\omega_1^3}, \tag{16}$$

$$\phi_3 = 1+54\frac{\omega_3}{\omega_1}+75\frac{\omega_5}{\omega_1}+2.3.9.25\frac{\omega_3\omega_5}{\omega_1^2}+3.9^3\left(\frac{\omega_3}{\omega_1}\right)^3+2.3.9.25^2\frac{\omega_3\omega_5^2}{\omega_1^3}, \tag{17}$$

$$\phi_5 = 1+\frac{50}{9}\frac{\omega_5}{\omega_3}+9\frac{\omega_3}{\omega_1}+2.9.25\frac{\omega_3\omega_5}{\omega_1^2}+\frac{25^3}{9}\frac{\omega_5^3}{\omega_1^2\omega_3}. \tag{18}$$

In first approximation it will be assumed that ω_3/ω_1 and ω_5/ω_1 will be so small that ϕ_1, ϕ_3, ϕ_5 can be put equal to 1. Then equation (13) gives for a load K (or λ) the relation between the imperfection β and the deflexion ω. Figure 2 depicts the left hand side of this

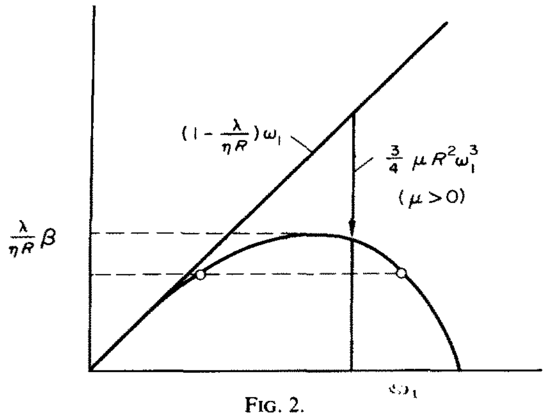

FIG. 2.

equation for $\mu > 0$. The ordinate being equal to $(\lambda/\eta R)\beta$ the figure enables to read for given values λ and β the deflexions ω_1 at which the column is in equilibrium. There are two positive roots ω_1. Keeping K (or λ) constant and increasing β the two roots approach each other and finally they coincide where $d\beta/d\omega_1 = 0$. This is the maximal imperfection at which equilibrium can exist at the load K, or inversely K is the maximal load that the column can sustain at this particular imperfection. The equation

$$d\beta/d\omega_1 = 0 \tag{19}$$

together with equation (13) yield the relation between β and the failure load K. Equation (19) yields the deflexion at the maximal load

$$\omega_1 = \frac{2}{3}\left(\frac{1-\lambda/\eta R}{\mu}\right)^{\frac{1}{2}}\cdot\frac{1}{R}, \tag{20}$$

whereupon equation (13) yields the imperfection β at which the column strength is K (or λ).

$$\beta = \frac{4}{9}\frac{\eta(1-\lambda/\eta R)^{\frac{3}{2}}}{\lambda\mu^{\frac{1}{2}}} \tag{21}$$

Having established ω_1 equations (17) and (18) yield

$$\frac{\omega_3}{\omega_1} = \frac{1-\lambda/\eta R}{9(9-\lambda/\eta R)}, \tag{22}$$

$$\frac{\omega_5}{\omega_1} = \frac{(1-\lambda/\eta R)^2}{3(9-\lambda/\eta R)(25-\lambda/\eta R)}. \tag{23}$$

Since $\lambda/\eta R$ is always smaller than 1, it appears that $\omega_3/\omega_1 < \frac{1}{81}$ and $\omega_5/\omega_1 < \frac{1}{675}$. Then equations (16–18) yield $\phi_1 < 1{\cdot}147, \phi_3 < 1{\cdot}807, \phi_5 < 1{\cdot}786$. Taking ϕ_1, ϕ_3, ϕ_5 into account the formulae (20–23) inclusive are to be replaced by

$$\omega_1 = (\omega_1)_1 \phi_1^{-\frac{1}{2}}, \tag{24}$$

$$\beta = \beta_1 \phi_1^{-\frac{1}{2}}, \tag{25}$$

$$\frac{\omega_3}{\omega_1} = \left(\frac{\omega_3}{\omega_1}\right)_1 \frac{\phi_3}{\phi_1}, \tag{26}$$

$$\frac{\omega_5}{\omega_1} = \left(\frac{\omega_5}{\omega_1}\right)_1 \frac{\phi_3 \phi_5}{\phi_1^2}, \tag{27}$$

where the quantities $(\ )_1$ refer to the first approximation given by equations (20) to (23) inclusive.

Taking for ϕ_1, ϕ_3, ϕ_5 their upper limits given above a second approximation of ω_1, β, ω_3/ω_1, ω_5/ω_1 is being obtained. Then equations (16–18) yield a second approximation of ϕ_1, ϕ_3, ϕ_5. After the fifth iteration the error is within 1 per cent with the following results $\phi_1 = 1{\cdot}446$, $\phi_3 = 3{\cdot}047$, $\phi_5 = 2{\cdot}455$.

$$\omega_1 > 0{\cdot}83(\omega_1)_1, \quad \beta > 0{\cdot}83(\beta)_1, \quad \frac{\omega_3}{\omega_1} < 2{\cdot}11\left(\frac{\omega_3}{\omega_1}\right)_1 = 0{\cdot}026, \quad \frac{\omega_5}{\omega_1} < 3{\cdot}58\left(\frac{\omega_5}{\omega_1}\right)_1 = 0{\cdot}0053.$$

These figures represent the lower limit for ω_1, and β and the upper limit for ω_3/ω_1 and ω_5/ω_1, which are reached when $\lambda/\eta R = 0$. They are being approached when $R \gg 1$.

The error when putting $\phi_1 = \phi_2 = \phi_3 = 1$ comes predominantly from ω_3/ω_1. It appears from equation (22) that the error decreases with $\lambda/\eta R$ increasing, therefore when R is small and λ is large. Then, as appears from equation (21), β will be small.

So as to give a quantitative idea of the magnitude of the error, two numerical examples will be mentioned.

1. $\alpha = 2{\cdot}5\%_0$, $R = 3{\cdot}29$, $\lambda/\eta R = 0{\cdot}297$, yielding $\beta_1 = 0{\cdot}116$.

   Then

$$\left(\frac{\omega_3}{\omega_1}\right)_1 = 0{\cdot}00897, \qquad \left(\frac{\omega_5}{\omega_1}\right)_1 = 0{\cdot}000767.$$

   Iteration yields

$$\phi_1 = 1{\cdot}193, \qquad \phi_3 = 2{\cdot}02, \qquad \phi_5 = 1{\cdot}91 \quad \text{and} \quad \frac{\omega_3}{\omega_1} = 1{\cdot}70\left(\frac{\omega_3}{\omega_1}\right)_1,$$

$$\frac{\omega_5}{\omega_1} = 2{\cdot}71\left(\frac{\omega_5}{\omega_1}\right)_1, \qquad \beta = 0{\cdot}914\beta_1.$$

2. $\alpha = 2{\cdot}5\%_0$, $R = 1{\cdot}83$, $\lambda/\eta R = 0{\cdot}527$, yielding $\beta_1 = 0{\cdot}062$.

Then

$$\left(\frac{\omega_3}{\omega_1}\right)_1 = 0{\cdot}00609, \qquad \left(\frac{\omega_5}{\omega_1}\right)_1 = 0{\cdot}000352.$$

Iteration yields

$$\phi_1 = 1{\cdot}09, \quad \phi_3 = 1{\cdot}51, \quad \phi_5 = 1{\cdot}53 \quad \text{and} \quad \frac{\omega_3}{\omega_1} = 1{\cdot}39\left(\frac{\omega_3}{\omega_1}\right)_1,$$

$$\frac{\omega_5}{\omega_1} = 1{\cdot}94\left(\frac{\omega_5}{\omega_1}\right)_1, \qquad \beta = 0{\cdot}957\beta_1.$$

The aim of this investigation is to establish for a given configuration α, R and a given load λ the imperfection β at which λ is the failure load. Therefore the significant error when omitting the iteration is the error of the ratio β/β_1.

In the first numerical example this error is 8·6 per cent, in the second one 4·3 per cent. It is not worthwhile to perform the iterative correction of these errors, firstly, because the model under consideration is an idealized one serving to study the trends of the effects of geometrical imperfections and secondly, because the analysis contains still another inaccuracy, the one caused by the truncation of the Taylor series of P after the third order term. This inaccuracy increases with increasing curvature, therefore with increasing imperfection β. In the first numerical example, where $\beta_1 = 11{\cdot}6$ per cent, the maximal strain $\Delta\varepsilon$ due to deflexion is $\Delta\varepsilon/\varepsilon_l = 21{\cdot}4$ per cent. In the second example, where $\beta_1 = 6{\cdot}2$ per cent, $\Delta\varepsilon/\varepsilon_l = 10{\cdot}5$ per cent. Whereas in this second case the truncated Taylor expansion reproduces the P-ε-relation up to $\Delta\varepsilon$ fairly well, this is no longer true in the first case. Then there is no need for the iterative correction with large imperfections and the small error associated with small imperfections should be disregarded. The conclusion is that in the numerical evaluation the approximation of β by equation (21) may be used.

3. EVALUATION AND DISCUSSION OF RESULTS

The numerical evaluation of β (equation 21), ω_3/ω_1 (equation 22) and of the maximal strain increments,

$$\frac{\varepsilon_2-\varepsilon_K}{\varepsilon_l} = R\omega_1\left(1+\frac{1-\lambda/\eta R}{9-\lambda/\eta R}\right)\left[1-\tfrac{1}{2}\eta' R\omega_1\left(1+\frac{1-\lambda/\eta R}{9-\lambda/\eta R}\right)\right],$$

$$\frac{\varepsilon_1-\varepsilon_K}{\varepsilon_l} = -R\omega_1\left(1+\frac{1-\lambda/\eta R}{9-\lambda/\eta R}\right)\left[1+\tfrac{1}{2}\eta' R\omega_1\left(1+\frac{1-\lambda/\eta R}{9-\lambda/\eta R}\right)\right],$$

have been carried out for α and are respectively 0·01 per cent, 1·25 per cent, 2·5 per cent, 5 per cent, 10 per cent, 20 per cent, with each α at a number of values of R from 0·7 up to about 4. With each set of values α, R a number of values $\lambda = K/K_l$ were taken, K being

TABLE 1

α%	0·01	0·25	2·5	5	10	20
λ_{max}	0·9989	0·9734	0·9577	0·9316	0·8916	0·8222

always smaller than the buckling load $K_b = \eta_b K_E$ of the perfectly straight column. Another upper limit of λ follows from the condition that $\mu > 0$: formula (20) and Fig. 2 show that with $\mu < 0$ the λ–ω curve has no maximum. These upper limits are function of α and are given in Table 1.

Figure 3 illustrates for one value of α (2·5 per cent) and for some values of R the shape of the β–λ curves. It shows that the value of R, where $K_b/K_l = 0{\cdot}9577$ ($\mu_b = 0$), separates two regions with different character of the β–λ curves. With smaller R any imperfection

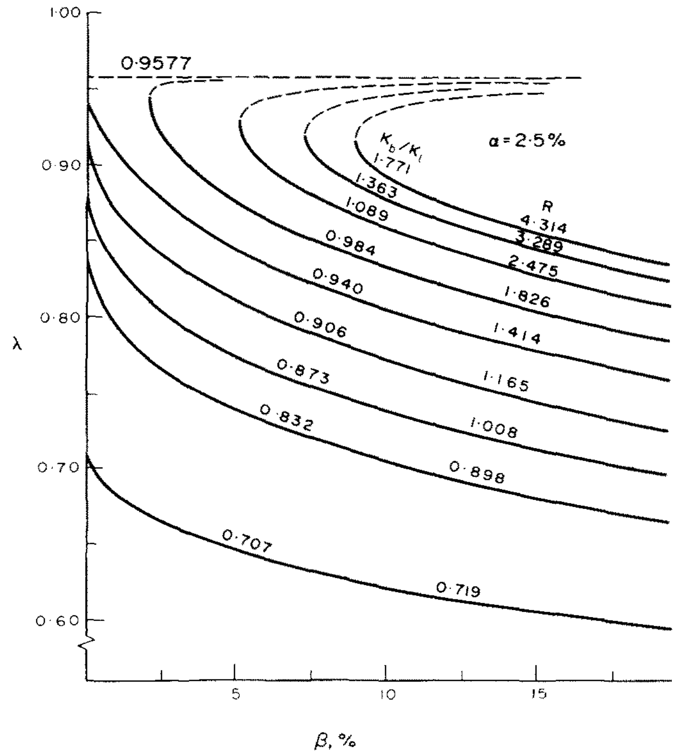

FIG. 3.

β yields reduction of the failure load λ as compared to the buckling load K_b/K_l of the perfectly straight column. When R is greater there is again reduction of strength provided β exceeds some critical value β_{cr}. Then the column strength does not even exceed K_l, though both K_E and K_b may be much greater than K_l. However for $\beta < \beta_{cr}$ the λ–ω curve has no other maximum than $\lambda_b = K_b/K_l$ at $\omega = \infty$ (and as usual at some finite value of ω failure will occur at a load somewhat below K_b). The width of this "window" β_{cr} across which the load carrying capacity can approach the high level of the buckling load K_b depends, as appears from Fig. 3, on R but as well on α.

Figure 4 gives β_{cr} as a function of R and α. With increasing R and α the "window" is wider, which means that the unfavourable effect of a certain amount of column imperfection is absent when R is large–short columns—and α is large–large imperfections of the flanges. This peculiar behaviour can be understood as follows.

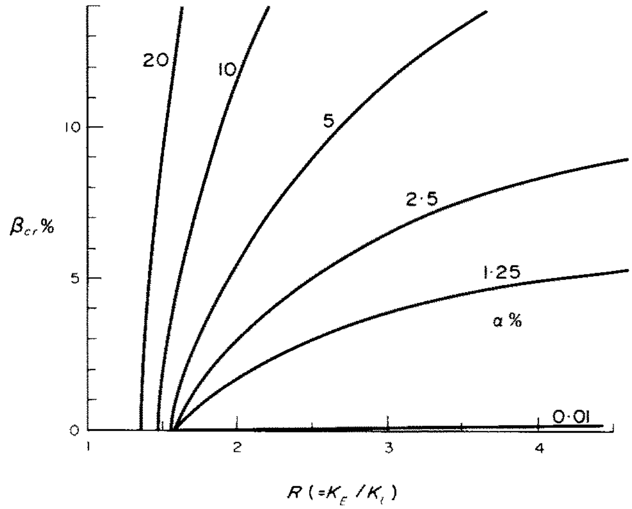

FIG. 4.

The column, before reaching the safe load level above K_l, where the imperfection hardly affects its strength, has to traverse the critical load range, where the second derivative of η (therefore $d^3P/d\varepsilon^3$) has large negative values (see Ref. [1], Fig. 5), which causes μ to be large. With small flange imperfections α this zone of danger where $-\eta''$ is large is situated near $K/K_l = 0{\cdot}8$ to $0{\cdot}9$. At this load level the imperfection β induces large deflexions. Only when β is sufficiently small (within the "window") the column can get across this zone of danger and reach the higher load level where a positive η'' exerts its favourable effect.

This phenomenon has much in common with the behaviour of rotating shafts. When the shaft is being accelerated slowly it will explode when traversing the critical speed, due to initial deflexion or eccentric masses.

A convenient representation of the numerical results is given by the Figs. 5a–e. They show λ as a function of R for a number of constant values of β. These curves have been obtained from graphs such as Fig. 3 for the various values of α.

The upward sweep of the curve for constant β towards the K_b/K_l curve ($\beta = 0$) occurs at the value of R at which $\beta = \beta_{cr}$. This value of R can be read from Fig. 4.

The validity of the results, shown in these figures, depends on the validity of the truncated Taylor series in representing the P–ε relation. This can be judged by considering the maximal strain increment $(\varepsilon_2 - \varepsilon_K)\varepsilon_l$ at the maximum load K.

The variation of slope of the P–ε curve near $\lambda = 1$ is less pronounced with increasing α. Therefore the range of validity of the truncated series increases with increasing α. Also the approximation is better when $1-\lambda$ increases. Some checks have been made; Table 2 gives the global limit up to which the truncated series represents $(\varepsilon - \varepsilon_K)/\varepsilon_l$ reliably.

The strain increment increases of course with increasing β, but as well with increasing R. Therefore it should be possible to give for each value of α the limit $\beta(R)$ up to which the curves are fairly correct. Such evaluation of the numerical results has not been carried out in full, but only partially. A practical upper limit for non-straightness of compression members is about $\beta = 6$ per cent. Then up to $R = 1{\cdot}8$ the maximal strain increment is within the limits, mentioned in Table 2, for α respectively 1·25, 2·5 and 5 per cent. As stated

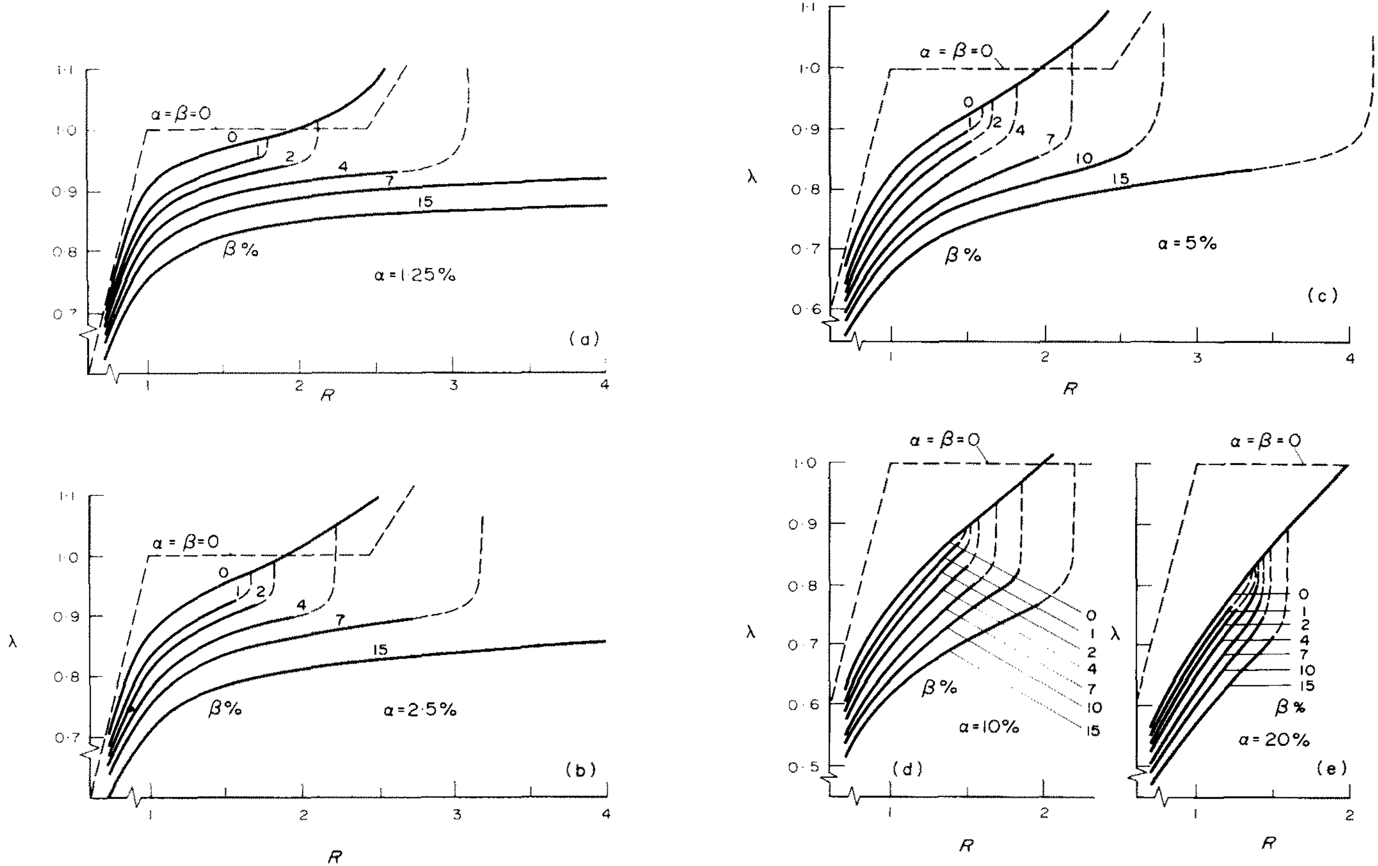

FIG. 5.

TABLE 2

α%	λ	$(\varepsilon-\varepsilon_K)/\varepsilon_l$
1·25	0·85	0·11
2·5	0·85	0·12
5	0·80	0·15
10	0·72	0·27

before the accuracy improves with greater α. For instance with $\alpha = 5$ per cent and $R = 1{\cdot}8$ the curves are correct up to $\beta = 10$ per cent. For $\alpha = 10$ per cent the curve for $\beta = 15$ per cent is correct up to $R = 1{\cdot}8$. Beyond those limits the general trend of the curves is correct, though they may be more or less in error numerically.

Inspection of Figs. 5 shows that the strength reduction $\lambda_b - \lambda$ due to β is (see Table 3):

1. Virtually constant over the range $0{\cdot}9 < R < 1{\cdot}6$;
2. Almost equal for α is 1·25 per cent, 2·5 per cent, 5 per cent and 10 per cent.

The reduction $1-\lambda_b$ of the buckling load due to the imperfection α alone is maximal at $R = 1$. A realistic value of α for actual structures is presumably 2 or 3 per cent. Figure 5b gives at $R = 1$ $1-\lambda_b = 0{\cdot}13$ and for $\beta = 7$ per cent $\lambda_b - \lambda = 0{\cdot}115$. At $R \neq 1$ $1-\lambda_b$ decreases rapidly and the strength reduction due to β predominates. It appears that both the α- and β-imperfections affect the failure load significantly.

4. CONCLUSIONS

Whereas initial waviness of the composing thin walls (α) reduces the buckling load mainly when $K_E/K_l\,(=R)$ is about 1, the imperfection of the column axis has its strength reducing effect over a much wider range of R, more in particular in the region $R > 1$ (Figs. 5).

It keeps the failure load K below K_l ($\lambda < 1$) up to a value R which depends on the

TABLE 3

R		0·9	1·0	1·2	1·4	1·5	1·6
β%	α%			$\lambda_b - \lambda$			
	1·25	0·075	0·085	0·085	0·08	0·08	0·08
	2·5	0·085	0·09	0·085	0·085	0·085	0·085
4	5	0·08	0·085	0·085	0·085	0·08	0·08
	10	0·06	0·07	0·07	0·07	0·065	—
	20	0·05	0·055	0·055	0	—	—
	1·25	0·10	0·115	0·11	0·105	0·105	0·105
	2·5	0·11	0·115	0·115	0·115	0·12	0·12
7	5	0·11	0·115	0·12	0·12	0·12	0·12
	10	0·10	0·105	0·105	0·11	0·11	0·11
	20	0·075	0·08	0·08	0·075	—	—
	1·25	0·13	0·15	0·15	0·145	0·145	0·145
	2·5	0·15	0·155	0·16	0·165	0·17	0·17
15	5	0·155	0·165	0·17	0·18	0·185	0·19
	10	0·145	0·15	0·165	0·18	0·19	0·20
	20	0·12	0·13	0·14	0·15	0·155	—

amount of the imperfections α and β. This limiting R-value increases with decreasing α and increasing β (Figs. 5). For a given set of values R and α there exists a critical imperfection β_{cr} (Fig. 4). When $\beta > \beta_{cr}$ λ does not exceed 1. When $\beta < \beta_{cr}$ the failure load is hardly affected by β and will be close to the buckling load K_b (for $\beta = 0$) (Fig. 3), which may involve $\lambda > 1$. β_{cr} increases with increasing α. Therefore it might have a beneficial effect upon the strength of columns when $R > 1{\cdot}5$ to apply artificial waviness of the walls with $\alpha > 0{\cdot}1$. The strength reduction $\lambda_b - \lambda$ due to β is almost constant over $0{\cdot}9 < R < 1{\cdot}6$ and is almost independent of α over $0{\cdot}0125 < \alpha < 0{\cdot}05$, which is the range of wall imperfection with actual structures (Table 3).

Taking the practical limit of β with actual structures to be 6 or 7 per cent it appears that α and β yield about equal strength reductions near $R = 1$, whereas the reduction due to β is the more important one when $R > 1$. The numerical results apply to the column section of Fig. 1, which is highly susceptible to the unfavourable effect of imperfections. Actual structures will be imperfection sensitive as well, though to a lesser degree, because by column bending the edges of some of their composing walls get strain increments of opposite sign and therefore will be less sensitive to wall imperfection.

The foregoing conclusions are restricted to elastic behaviour.

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APPENDIX

The nonlinear relation between bending moment and column axis curvature

Reference 1 raises the question: how must the load vary so as to maintain equilibrium when the column deflects slightly at the bifurcation load K_b? This equilibrium load

$$K = K_b + k, \tag{1}$$

where k is quadratic in the deflexion W.

The flange loads at K_b are $P_b = \frac{1}{2}K_b$; with the deflected column the upper and lower flange carry P_1 and P_2, respectively. Then

$$K_b + k = P_1 + P_2, \tag{2}$$

$$M = (P_1 - P_2)c. \tag{3}$$

The longitudinal flange edges are simply supported by webs, which serve to maintain structural integrity of the column without contributing to the transmission of axial stresses.

The compressive strains of the flange edges are ε_b at K_b and under deflexion ε_1, ε_2 respectively. They are related to the deflexion W by the geometric relation

$$\frac{\varepsilon_1 - \varepsilon_2}{2c} = \frac{\mathrm{d}^2 W}{\mathrm{d}x^2}. \tag{4}$$

Using the Taylor series for the relation between P and ε, truncating it after the third order term, we have

$$P_i = P_b + \left(\frac{\mathrm{d}P}{\mathrm{d}\varepsilon}\right)_b (\varepsilon_i - \varepsilon_b) + \frac{1}{2}\left(\frac{\mathrm{d}^2P}{\mathrm{d}\varepsilon^2}\right)_b (\varepsilon_i - \varepsilon_b)^2 + \frac{1}{6}\left(\frac{\mathrm{d}^3P}{\mathrm{d}\varepsilon^3}\right)_b (\varepsilon_i - \varepsilon_b)^3 \qquad i = 1, 2. \tag{5, i}$$

The five equations (2–5) contain seven unknown quantities:

$$\varepsilon_1, \varepsilon_2, P_1, P_2, W, k \text{ and } M.$$

Using these five equations for eliminating the four unknowns ε_1, ε_2, P_1, P_2 the result is a relation between M, W and k:

$$M = \eta_b EI \frac{\mathrm{d}^2W}{\mathrm{d}x^2}\left[1 - \tfrac{1}{6}(2\eta'^2 - \eta\eta'')_b \left(\frac{c}{\varepsilon_l}\right)^2 \left(\frac{\mathrm{d}^2W}{\mathrm{d}x^2}\right)^2 + \left(\frac{\eta'}{\eta}\right)_b \frac{k}{K_l}\right], \tag{6}$$

where

$$\eta = \frac{\mathrm{d}P/P_l}{\mathrm{d}\varepsilon/\varepsilon_l}, \qquad (\;)' = \mathrm{d}(\;)/\mathrm{d}(P/P_l)$$

and I is the moment of inertia of the section $I = 2bc^2h$.

Recalling that k is quadratic in W equation (6) says that deflexion modifies the bending stiffness $\eta_b EI$ according to the expression between square brackets, which is a quadratic function of W and thereby a function of x.

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Абстракт—В ссылке /1/ указывается чувствительность к неточности тонкостенного сжимаемого элемента к начальной волнистости составных стенок и к неточности оси колонны. Однако, эффект последней неточности не анализирован удовлетворительно. Предлагаемая статья дает исправление и определяет, для случая идеализированной колонны, понижение нагрузки разрушения, в следствии этого рода неточности, которая оказывается значительной. Предполагается поведение в упругой области.